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JUNCTION CAPACITANCE AS A FUNCTION OF VOLTAGE FOR DIFFUSED p-n DIODES WITH EXPONENTIAL DOPING GRADIENTS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1971

1. Report No. NASA TM X-2179	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle JUNCTION CAPACITANCE AS A FUNCTION OF VOLTAGE FOR DIFFUSED p-n DIODES WITH EXPONENTIAL DOPING GRADIENTS		5. Report Date February 1971 6. Performing Organization Code
7. Author(s) Ira T. Myers and Julian F. Been		8. Performing Organization Report No. E-5931
		10. Work Unit No. 120-60
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Exponential doping p-n junction Capacitance Diodes	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	13	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

A method was developed for calculating the capacitance of a diffused p-n junction as a function of voltage that can be used in place of the more complicated methods now available. A single universal curve, based on an exponential doping gradient, permits the calculation of the capacitance of junctions with a wide variety of base doping levels and diffusion doping gradients. In particular, the method permits the calculation of capacitances at intermediate voltages where the junction behaves neither as a linear graded junction nor as an abrupt junction, but is intermediate between the two cases. Calculated and measured capacitances as a function of voltage were compared for a silicon power diode with good agreement.

INTRODUCTION

The capacitances of linearly graded and abrupt junctions are treated in many textbooks (refs. 1 and 2). A reverse biased diffused p-n junction will normally show a $(V - V_B)^{-1/3}$ dependence of capacitance at low voltages and a $V^{-1/2}$ dependence at high voltages. The "built-in" or diffusion voltage of the junction is V_B.

Between the low and high voltages a transition region exists where the capacitance fits neither the linear graded $(V - V_B)^{-1/3}$ nor the abrupt $(V)^{-1/2}$ case. Lawrence and Warner (ref. 3) calculated this case using the complementary error function and the gaussian distribution as doping distributions. This report describes a similar calculation using an exponential doping distribution. Justification is given for using the exponential doping distribution results for all diodes, no matter what the doping distribution actually is. The advantages of using the exponential doping results, as given in this report, are shown to be improved clarity of interpretation of the doping gradient, and a considerably more straightforward calculation procedure. This report concludes with a comparison of calculated and measured capacitances as a function of voltage for a silicon power diode (ref. 4).

CALCULATION OF CAPACITANCE AGAINST VOLTAGE FOR A DIFFUSED p-n JUNCTION

Consider a junction where the dopant has been added such that the added impurity follows an exponential distribution. For example, take p type base material, with a concentration of N_o dopant atoms per cubic centimeter. The concentration N_D of the n type dopant atoms is given by

$$N_D = N_s e^{-bX} \tag{1}$$

where N_s is the surface concentration, X the distance from the surface, and b a constant. The net doping $N_D - N_o$ is given by

$$N = N_s e^{-bX} - N_o \tag{2}$$

At the junction $N = 0$ and $X = X_o$ so that

$$N_o = N_s e^{-bX_o} \tag{3}$$

(see fig. 1). The measured capacitance C of the junction is

$$C = \frac{dQ}{dV} \tag{4}$$

where V is measured in the reverse direction. The capacitance C is also given by

$$C = \frac{\epsilon\epsilon_o A}{W} \tag{5}$$

where

$$W = X_2 - X_1 \tag{6}$$

and where W is the depletion region width, $\epsilon\epsilon_o$ the permittivity, and A the area of the junction.

With applied voltage the depletion region expands an amount dX_1 in the n region ($X_1 \rightarrow X_1 - dX_1$) and an amount dX_2 in the p region ($X_2 \rightarrow X_2 + dX_2$). The amount of positive charge uncovered by the expansion of the depletion region in the n region is

$$dQ_n = qA\left(N_s e^{-bX} - N_o\right)(dX_1) \tag{7}$$

Similarly, the amount of charge uncovered by the expansion of the depletion region in the p type base is

$$dQ_p = qA\left(N_o - N_s e^{-bX_2}\right)dX_2 \tag{8}$$

In order to maintain charge neutrality over the entire depletion region,

$$dQ_n = dQ_p \tag{9}$$

The total dW is given by

$$dW = |dX_2| + |dX_1| \tag{10}$$

Next, equations (4), (5), and (7) to (10) are used to eliminate dQ_n and dQ_p and to express dW in terms of dV:

$$dW = \frac{\epsilon\epsilon_o}{qW}\left(\frac{1}{N_o - N_s e^{-bX_2}} + \frac{1}{N_s e^{-bX_1} - N_o}\right)dV \tag{11}$$

This is a key equation in the development. It will be integrated to give W and therefore C as a function of V.

Before equation (11) can be integrated, X_1 and X_2 must be expressed in terms of W. The quantity X_2 is easily expressed in terms of X_1 and W as

$$X_2 = W + X_1 \tag{12}$$

so that it is only necessary to find the relation between X_1 and W. This is obtained by equating the positive charge on the n side of the junction to the negative charge on the p side of the junction:

$$qA \int_{X_1}^{X_o} \left(N_s e^{-bX} - N_o\right) dX = qA \int_{X_o}^{X_1+W} \left(N_o - N_s e^{-bX}\right) dX \tag{13}$$

After integration and simplification the expression for X_1 is

$$e^{-bX_1} = \frac{bN_oW}{(1 - e^{-bW})N_s} \tag{14}$$

Substituting equations (14) and (12) into equation (11) gives

$$dW = \frac{\epsilon\epsilon_o}{qN_oW} dV \left[\frac{1 - e^{-bW}}{1 - e^{-bW} - bWe^{-bW}} + \frac{1 - e^{-bW}}{bW - 1 + e^{-bW}}\right] \tag{15}$$

where dV is positive for increasing reverse voltage.

As stated previously, W and V are variables of integration. Introducing dummy variables of integration (Ω for bW, and φ for V) allows the use of W and V for integration limits. After some simplification,

$$\left[\frac{\Omega(1 + e^{-\Omega})}{1 - e^{-\Omega}} - \frac{\Omega^2 e^{-\Omega}}{(1 - e^{-\Omega})^2} - 1\right] d\Omega = \frac{b^2\epsilon\epsilon_o}{qN_o} d\varphi \tag{16}$$

This expression was integrated over the depletion region. The limits for Ω were $\Omega = 0$ and $\Omega = bW$, where W is the depletion region width at a voltage V. The corresponding limits for φ are $-V_B$ (corresponding to $W = 0$) and the applied voltage V. Therefore,

$$\int_0^z \left[\frac{\Omega(1 + e^{-\Omega})}{(1 - e^{-\Omega})} - \frac{\Omega^2 e^{-\Omega}}{(1 - e^{-\Omega})^2} - 1\right] d\Omega = \frac{b^2\epsilon\epsilon_o}{qN_o} \int_{-V_B}^{V} d\varphi \tag{17}$$

where the substitution $z = bW$ has been made. This may be easily integrated by integrating the second term in the first integral,

$$\int_0^z \frac{\Omega^2 e^{-\Omega}}{(1 - e^{-\Omega})^2} d\Omega$$

by parts and by combining the results with the other terms prior to integrating them. The result is

$$\frac{z^2}{1 - e^{-z}} - \frac{z^2}{2} - z = \frac{b^2 \epsilon\epsilon_o (V + V_B)}{qN_o} \tag{18}$$

Define the function $G(z)$ as

$$G(z) = \frac{+z^2}{1 - e^{-z}} - \frac{z^2}{2} - z \tag{19}$$

where $z = bw$ as defined previously. The function $G(z)$ is always positive. Thus

$$G(z) = \frac{b^2 \epsilon\epsilon_o (V + V_B)}{qN_o} \tag{20}$$

Then

$$z = bw = G^{-1}\left(\frac{b^2 \epsilon\epsilon_o (V + V_B)}{qN_o}\right) \tag{21}$$

where G^{-1} is the inverse function of G. It was not possible to find the inverse of G and express it in closed form algebraically (i.e., an expression $f(G) = z$, which would be the solution for z of eq. (19)). Instead, G was plotted against z (fig. 2) and tabulated (see table I) so that for any given value of G the value of z may be determined.

In the range $0.1 \leq G \leq 10$, the quantity z (or bw) can be represented, within ± 5 percent, as

$$z = 1.80\ G^{1/3} + 0.60\ G^{1/2} \tag{22}$$

where $G = b^2\epsilon\epsilon_o(V + V_B/qN_o$. For $G \leq 0.1$,

$$z = (12\,G)^{1/3} \tag{23}$$

is a close approximation, while for $G > 10$,

$$z = 1 + \sqrt{1 + 2G} \tag{24}$$

can be used.

The capacitance against voltage can now be directly calculated:

$$C = \frac{\epsilon\epsilon_o Ab}{z} \tag{25}$$

where the quantity z contains the voltage dependence. For the limiting case of low voltages, where $z = (12G)^{1/3}$, the capacitance reduces to the expected cube root dependence

$$\frac{C}{A} = \left[\frac{\epsilon^2\epsilon_o^2 qN_o b}{12(V + V_B)}\right]^{1/3} \tag{26}$$

Similarly at high reverse voltages, where $z = 1 + \sqrt{1 + 2G} \approx \sqrt{2G}$,

$$\frac{C}{A} = \left[\frac{\epsilon\epsilon_o qN_o}{2(V + V_B)}\right]^{1/2} \tag{27}$$

which is the expected square root dependence.

In summary, to calculate the capacitance at a given voltage V, first calculate $b^2\epsilon\epsilon_o(V + V_B)/qN_o$, then find z from figure 2 or table I corresponding to it, and finally calculate C from

$$C = \frac{\epsilon\epsilon_o bA}{z} \tag{28}$$

If V_B is not known, it can be calculated from the work of Nuyts and Van Overstraeten (ref. 4). A fit to their results gives

$$V_B = 0.080 \log_{10} bN_o - 1.08 \tag{29}$$

in the range of bN_o between 10^{17} and 10^{23} (bN_o is the doping gradient in cm^{-4} at the junction).

SAMPLE CALCULATION

To illustrate the method, including the units to be used, a sample calculation is given. Consider a diode with the following characteristics:

A	junction area, 0.1 cm^2
V	applied voltage, 3.0 V
b	logarithmic doping gradient, 10^4 cm^{-1}
N_o	base doping level, $10^{15}/cm^3$
bN_o	doping gradient at junction, 10^{19} cm^{-4}

Since V_B is not known, calculate it from the Nuyts and Van Overstraeten expression

$$V_B = 0.08 \log_{10} bN_o - 1.08 = 0.44 \text{ V}$$

Then

$$G = \frac{10^8(1.06\times10^{-12})(3.0 + 0.44)}{1.602\times10^{-19}(10^{15})} = 2.28$$

and z = 3.2 from figure 2. From this value of z

$$C = \frac{\epsilon\epsilon_o bA}{z} = \frac{(1.06\times10^{-12})(10^4)(0.1)}{3.2} = 330\times10^{-12} \text{ F or 330 pF}$$

COMPARISON OF CALCULATED AND MEASURED RESULTS

The technique just described was used to calculate the capacitance as a function of voltage for a p-i-n type diode number SIN1189 (ref. 5). This was a 35-ampere diode with a junction area of 0.455 square centimeter and a base doping level of about 1.2×10^{14} cubic centimeters. The value of b was about 5×10^{3} per centimeter.

For the comparison, N_0 was determined from experimental measurements of capacitance by fitting at the high voltage end of the curve, and b was determined by fitting at the low voltage end. The N_0 as determined for the diode used was 1.1×10^{14} per cubic centimeter and b was 5.1×10^{3} per centimeter. The diffusion voltage V_B was determined both by extrapolation of a $1/C^3$ plot against voltage, and from b using Nuyts and Van Overstraeten's work. The value of V_B as determined graphically from the $1/C^3$ plot was 0.35 volt; the calculation from Nuyts and Van Overstraeten's work gave 0.34 volt.

Figure 3 gives a comparison between calculated values and measured values of capacitance. The agreement is good.

Table II gives a comparison of experimental results with the capacitances as calculated using the three different types of doping gradients (exponential, from this report, and complimentary error function and gaussian from Lawrence and Warner's work (ref. 3)). To compare the goodness of fit for the three doping gradients used, the ratio $C_{calculated}/C_{measured}$ was averaged over the voltage points from 1 to 100 volts. For a perfect fit this ratio should be 1.00. The value of this ratio, and its error, give an estimate of how the model agrees with experiment (see table III). Ninety percent confidence limits are used on the errors.

The curves were fitted at zero bias and at 200 volts in each case. There is less difference between the calculations using different types of gradients than between calculated and experimental values. The particular kind of doping gradient assumed does not make much difference in the calculated capacitance. The variation in calculated capacitance from one doping gradient type to another is within the calculated errors. This could be expected, since at very low biases, with a narrow depletion region, any gradient can be fitted with a straight line. On the other hand, at high voltages, nearly the entire depletion region is in the base, so that again the type of gradient is relatively unimportant. Therefore, the simplest to use gradient, namely the exponential, may be used under a wide range of conditions.

SUMMARY AND CONCLUSIONS

A method of calculating capacitance as a function of voltage for an exponentially doped p-n junction is described. The method is significantly easier to use than presently available methods where a complementary error function or gaussian doping gradient is assumed. Comparison with a power diode gave good agreement with experiimental results. Calculated capacitances using exponential, complementary error function, and gaussian distribution all gave results in agreement with measured values within the calculated errors. This indicates that the dopant distribution assumed is not critical, and that the more convenient and easier to interpret exponential distribution can be used for capacitance against voltage calculations.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 7, 1970,
120-60.

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TABLE I. - NUMERICAL VALUES OF $G(z)$ FOR VALUES OF z FROM 0 TO 9.9

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0	8.3319×10^{-5}	6.6622×10^{-4}	2.2466×10^{-3}	5.3192×10^{-3}	1.037×10^{-2}	1.7893×10^{-2}	2.8353×10^{-2}	4.2218×10^{-2}	5.9945×10^{-2}
1	8.1977×10^{-2}	1.0874×10^{-1}	1.4066×10^{-1}	1.7813×10^{-1}	2.2153×10^{-1}	2.714×10^{-1}	3.2760×10^{-1}	3.9096×10^{-1}	4.6163×10^{-1}	5.3990×10^{-1}
2	6.2607×10^{-1}	7.2039×10^{-1}	8.2311×10^{-1}	9.3447×10^{-1}	1.0547	1.1839	1.3224	1.4702	1.6276	1.7947
3	1.9716	2.1583	2.3551	2.5620	2.7791	3.0064	3.2441	3.4920	3.7504	4.0192
4	4.2985	4.5883	4.8885	5.1993	5.5206	5.8525	6.1949	6.5478	6.9112	7.2851
5	7.6696	8.0645	8.4670	8.8859	9.3123	9.7491	10.196	10.654	11.122	11.601
6	12.089	12.589	13.098	13.618	14.148	14.689	15.239	15.800	16.372	16.953
7	17.545	18.147	18.759	19.381	20.013	20.656	21.309	21.972	22.645	23.328
8	24.021	24.725	25.438	26.162	26.896	27.640	28.394	29.158	29.932	30.716
9	31.510	32.314	33.129	33.953	34.787	35.632	36.486	37.351	38.225	39.110

TABLE II. - COMPARISION OF THREE METHODS OF CALCULATION OF CAPACITANCE WITH EXPERIMENT

Applied voltage	Calculated capacitances			Experimental capacitance,
	C_{erfc}	C_{gauss}	C_{expon}	C_{exper}
0	1300	1300	1300	1300
1	840	770	790	840
2	640	640	645	680
5	480	470	466	490
10	380	360	357	375
20	270	270	270	290
50	190	180	181	190
100	137	136	131	132
200	95	95	95	95

TABLE III. - RATIO OF CALCULATED TO MEASURED CAPACITANCES FOR DIFFERENT DOPING GRADIENTS

Type of doping gradient	Average value of C_{calc}/C_{meas}
Complimentary error function	0.99±0.08
Gaussian	0.96±0.07
Exponential	0.95±0.04

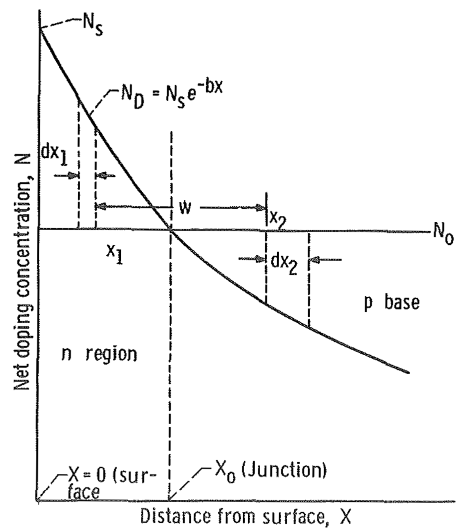

Figure 1. - Doping concentration.

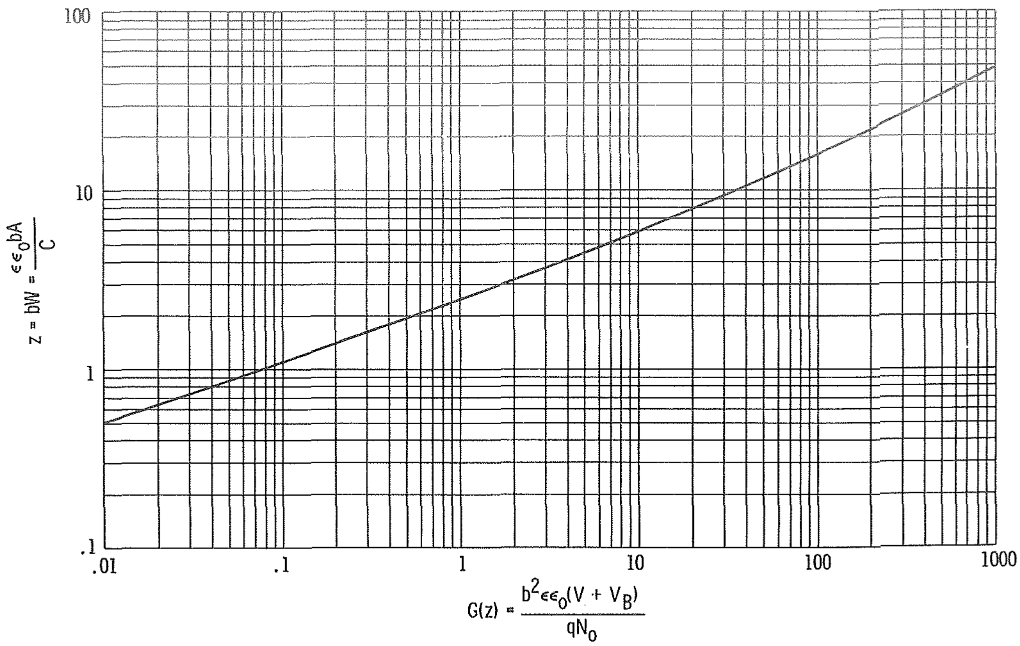

Figure 2. - Inverse junction capacitance function z versus the applied voltage function G.

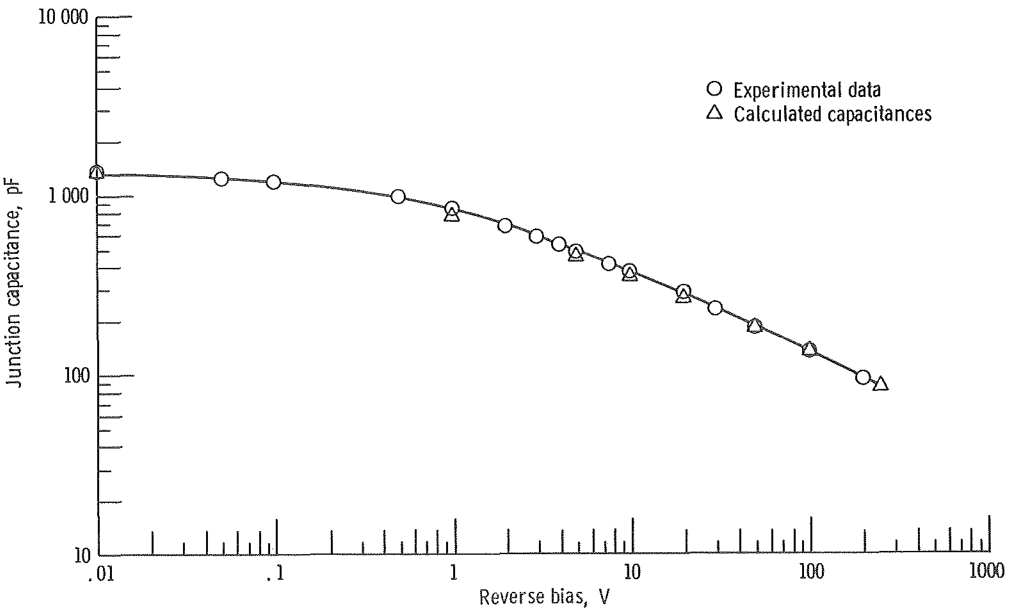

Figure 3. - Junction capacitance as function of reverse bias for diode 748.

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